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MODELING OF ROLLED-SHEET COOLING

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Stationary radiative and convective heat transfer on an infinite moving plate is studied numerically. A conjugate formulation of the problem allows one to correctly take into account the interaction between temperature fields of the plate and the surrounding gas medium. The determining influence of radiation on the formation of plate temperature is established. The effect of different regime factors on heat transfer is analyzed.

The optimization of metal heat treatment and the creation of energy saving technologies in metallurgy require allowance for many factors in mathematical simulation of the corresponding heat transfer processes [1, 2]. In particular, the metallurgical quality of rolled steel is mainly determined by the temperature difference inside the blank during treatment. A change in temperature of 1% can lead to a 10%-decrease in yield strength, which in turn, will cause inhomogeneity of the strength properties of the final product [3]. The quality of thermal calculations in the traditional approach is to a considerable extent determined by knowledge of the coefficient of heat transfer. However, in the majority of practical cases its accurate determination can be impossible [4, 5]. Thus, the need arises to correctly analyze the heat transfer of a moving rolled sheet with the surrounding medium with allowance for convection and radiation. In this case, due to the mentioned problematic character of employment of the coefficient of heat transfer in calculations, it seems reasonable to consider the problem in a conjugate formulation, i.e., when the surface temperature is not *a priori* given but is determined in problem solution.

We consider radiative and convective heat transfer in a conjugate formulation of the problem when cooling of a $2H$-thick plane plate being drawn from a slot at constant velocity u_0 into a motionless gas medium (Fig. 1). Due to the viscosity of the medium, a boundary layer is formed on the surface of the moving plate. In contrast to a classical boundary layer, the velocity distribution in this case is different: on the plate surface (at $y = 0$) the longitudinal velocity component is equal to the velocity of plate motion $u = u_0$ and at an infinite distance, where the condition of a stationary medium holds $u = 0$.

The system of equations describing heat and mass transfer processes in the boundary layer for a medium with constant thermophysical properties has a form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\,, \quad x > x_0\,, \quad 0 < y < \infty\,; \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}\,; \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = a \frac{\partial^2 T}{\partial y^2}\,, \tag{3}$$

$x = x_0$: [illegible] (4)

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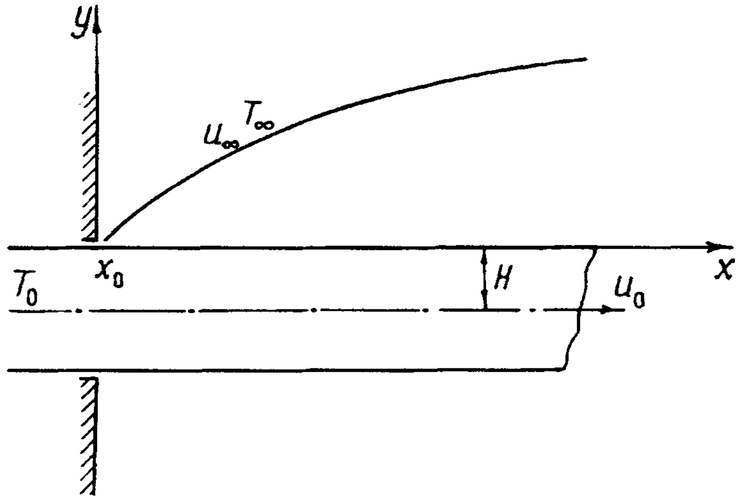

Fig. 1. Physical model and coordinate system.

$$y = 0: \quad u = u_0\,, \ \ v = 0\,, \ \ T = T_0\,; \tag{5}$$

$$y \to \infty: \quad u \to 0\,, \ \ T \to T_\infty\,. \tag{6}$$

Heat transfer on an infinitely moving plate is described by the equation

$$u_0 \frac{\partial T_s}{\partial x} = a_s \frac{\partial^2 T_s}{\partial y^2}\,, \ \ x > x_0\,, \ \ -H < y < 0\,, \tag{7}$$

with the boundary conditions

$$x = x_0: \quad T_s = T_0\,, \tag{8}$$

$$y = -H: \quad \frac{\partial T_s}{\partial x} = 0\,, \tag{9}$$

$$y = 0: \quad -\lambda_s \frac{\partial T_s}{\partial x} = -\lambda \frac{\partial T}{\partial x} + E\,. \tag{10}$$

Here

$$E = \frac{\sigma T_w^4 - \sigma T_\infty^4}{1/\varepsilon_w + 1/\varepsilon_\infty - 1}\,. \tag{11}$$

This problem was considered ignoring radiation for the first time in [6]. The mathematical model suggested in this work for describing convective heat transfer on a moving infinite surface has been repeatedly used in subsequent studies [7-10]. As was found in [10], such problems of heat transfer on a moving surface should be considered only in a conjugate formulation.

To numerically solve the boundary layer equations (1)-(6) it is convinient to use the self-similar variable

$$\eta = y \left(\frac{u_0}{\nu x}\right)^{1/2}\,. \tag{12}$$

as a transverse dimensionless coordinate.

The longitudinal dimensionless coordinate is defined as

$$\xi = \frac{x}{H\mathrm{Re}} . \tag{13}$$

In this case the dynamic portion of the problem is reduced to the Blasius differential equation

$$2f''' + ff' = 0 , \quad 0 < \eta < \infty , \tag{14}$$

but the boundary conditions at $\eta = 0$ and $\eta = \infty$, in contrast to the Blasius equation, change places:

$$\eta = 0: \quad f = 0 , \ f' = 1 ; \tag{15}$$

$$\eta = \infty: \quad f' = 0 . \tag{16}$$

Energy equation (3) with the boundary conditions (4)-(6) in the chosen dimensionless variables has the form

$$\mathrm{Pr}\, f' \xi \frac{\partial \theta}{\partial \xi} = \frac{\partial^2 \theta}{\partial \eta^2} + \mathrm{Pr} \frac{f}{2} \frac{\partial \theta}{\partial \eta} , \quad 0 < \eta < \infty , \ \xi_0 < \xi < \infty , \tag{17}$$

$$\xi = \xi_0: \quad \theta = \theta_0 ; \tag{18}$$

$$\eta = 0: \quad \theta = \theta_w , \ \eta \to \infty: \quad \theta \to \theta_\infty . \tag{19}$$

Heat transfer in the plate is described by the dimensionless relation

$$B\mathrm{Pr} \frac{\partial \theta_s}{\partial \xi} = \frac{\partial^2 \theta_s}{\partial \zeta^2} , \ \xi_0 < \xi < \infty , \ -1 < \zeta < 0 , \tag{20}$$

with boundary conditions

$$\xi = \xi_0: \quad \theta_s = 1 , \tag{21}$$

$$\zeta = -1: \quad \frac{\partial \theta_s}{\partial \xi} = 0 , \tag{22}$$

$$\zeta = 0: \quad \frac{\partial \theta_s}{\partial \zeta} = \kappa \left(\xi^{-1/2} \frac{\partial \theta}{\partial \eta} \bigg|_{\eta = 0} - \mathrm{Sk}\, \Phi \right) . \tag{23}$$

Here θ_0 is a self-similar solution of equation (17); $B = a_\infty / a_s$; $\Phi = E / 4\sigma T_0^4$ is the dimensionless density of the radiation flux; the prime denotes differentiation with respect to the variable η.

Problem (14)-(16) is solved numerically by the method of finite differences using the approach suggested in [11]. The reliability of the obtained results is confirmed by good agreement (within 1%) with similar results from [7].

Conjugate problems are more complex than problems in a traditional separate formulation and in each specific case they require their own mathematical means, which are usually based on numerical methods. In the given case, the general solution scheme assumes an iteration approach: the plate surface temperature is determined

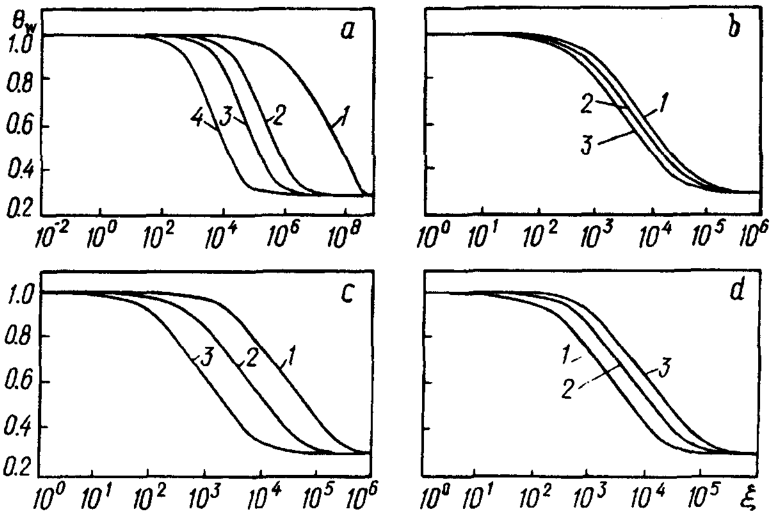

Fig. 2. The effect of emissivity ε (a) ($\chi = 10^{-3}$, $B = 5$, Sk = 3), Stark number Sk (b) ($\chi = 10^{-3}$, $B = 5$, $\varepsilon = 0.9$), conjugation parameter χ (c) ($B = 5$, Sk = 3, $\varepsilon = 0.9$), and parameter B (d) ($\chi = 10^{-3}$, Sk = 3, $\varepsilon = 0.9$) on the temperature of the plate surface. a: 1) $\varepsilon = 0$, 2) 0.02, 3) 0.1, 4) 0.9; b: 1) Sk = 2, 2) 3, 3) 4; c: 1) $\chi = 2 \cdot 10^{-4}$, 2) 10^{-3}, 3) $5 \cdot 10^{-3}$; d: 1) $B = 2$, 2) 5, 3) 10.

in section ξ; using this temperature as a boundary condition, the temperature distribution in the boundary layer and on the plate are found by equations (17) and (20); and a new approximation for the plate surface temperature is found by the "shooting" technique, where the conjugation condition (23) serves as a residual. Boundary-value problems (17)-(19) and (20)-(23) are solved by the method of finite differences by an implicit scheme with a second-order approximation.

As is seen from the formulation, Pr, Sk, B, χ, and ε_w are the determining parameters of the problem. For a metal–air system at temperatures of the order of $T_0 \simeq 1000$ K and plate thickness $H = 1$ mm, these parameters vary within the limits Pr $\simeq 0.7$, Sk $\simeq 2-4$, $B \simeq 2-10$, $\chi \simeq 0.0002-0.005$, $\varepsilon_w \simeq 0.02-0.9$. The temperature of the outer region was taken to be $\theta_\infty = 0.3$.

In Figs. 2a and 2b the effect of radiation on temperature is shown. Curves ⁻-4 in Fig. 2a refer to the case of combined radiative and convective cooling of the plate; curve 1 refers to the case of convective cooling alone. A comparison of these curves indicates the determining role of radiation in heat transfer within the given temperature range. Plate cooling to the ambient temperature due to radiative heat transfer takes place much closer to the point of drawing from the slot than does convective.

Under conditions of a prevailing radiation effect on the formation of the plate temperature field, the role of the optical properties of a medium and of the plate surface in heat transfer increases. As the plate surface emissivity increases, the temperature grows and/or the coefficient of thermal conductivity of the medium decreases (when the role of radiation in heat transfer increases compared to heat conduction, i.e., the Stark number increases), the process of plate cooling is noticeably enhanced (Figs. 2a and 2b, respectively).

Figure 2b presents the results of calculation of the plate surface temperature as a function of the parameter χ. The conjugation parameter χ was first introduced into the theory of heat transfer by A. V. Luikov. Its form depends on the specific formulation of the problem and the choice of dimensionless variables. In the general case, it can be represented as [5]

$$\kappa = \frac{\lambda L}{\lambda_s H} \Pr^n \mathrm{Re}^m , \tag{24}$$

where L is the plate length and m and n are the constants. The parameter χ characterizes the ratio of the heat flux escaping from the plate to the boundary layer to the heat flux spreading in the plate. If $\chi < \chi_*$, where χ_* is some critical value, then the problem can be studied in a separate formulation without regard for the temperature fields in the plate and gaseous medium. For highly intense heat transfer processes, which include radiative and convective heat transfer, the problem should, as a rule, be considered only in a conjugate formulation.

The presented results indicate a substantial dependence of the temperature distribution of the plate on the conjugation parameter. As χ increases, faster transition of the temperature curves to an equilibrium value is observed.

The parameter B characterizes the effect of longitudinal heat transfer with respect to transverse heat transfer in the plate, and the smaller is its value, the greater is the role of the transverse heat flux in the formation of plate temperature. Since plate cooling takes places from the surfaces, i.e., due to transverse heat transfer, a decrease in B leads to more intense plate cooling (Fig. 2d).

It follows from the analysis of the presented results of the study of radiative and convective heat transfer on an infinite moving plate that at high temperatures radiation plays a determining role in plate cooling; due to this fact the effect of the optical properties of the medium and surface on heat transfer increases.

Thus, correct modeling of these problems requires a more accurate allowance for the effect of optical factors on the processes of thermal interaction between a gaseous medium and a solid body, i.e., the problem formulation should include the equation of radiative transfer in an opaque medium with nonblack boundaries.

NOTATION

x, y, coordinates; u, v, velocity components; T, temperature; a, thermal diffusivity; E, density of radiation flow on the surface; λ, thermal conductivity; ν, kinematic viscosity; σ, Stefan–Boltzmann constant; f, dimensionless stream function; $\theta = T/T_0$, dimensionless temperature; ε, emissivity; $\eta = y(u_0/\nu x)^{1/2}$, $\zeta = y/H$, transverse dimensionless coordinates; $\xi = x/(H\mathrm{Re})$, longitudinal dimensionless coordinate; $\mathrm{Re} = u_0H/\nu$, Reynolds number; $\mathrm{Pr} = \nu_\infty/a_s$, Prandtl number; $\mathrm{Sk} = 4\sigma T_\infty^3 H/\lambda_\infty$, Stark number; $\chi = \lambda_\infty/\lambda_s$, conjugation parameter. Indices: s, solid body; w, plate surface; ∞, undisturbed region; 0, initial value.

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